Chapter 10
CIRCLES

MODULE - 3/3

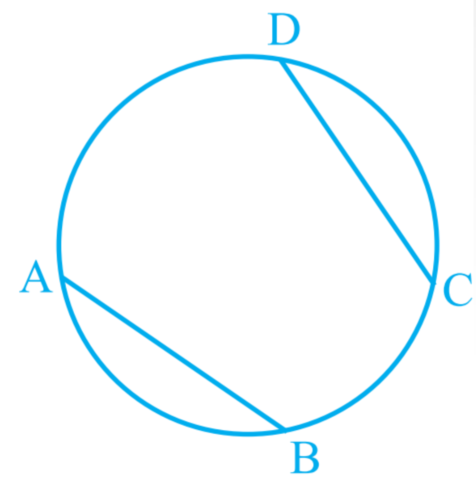
CIRCLES

In this module we learn about

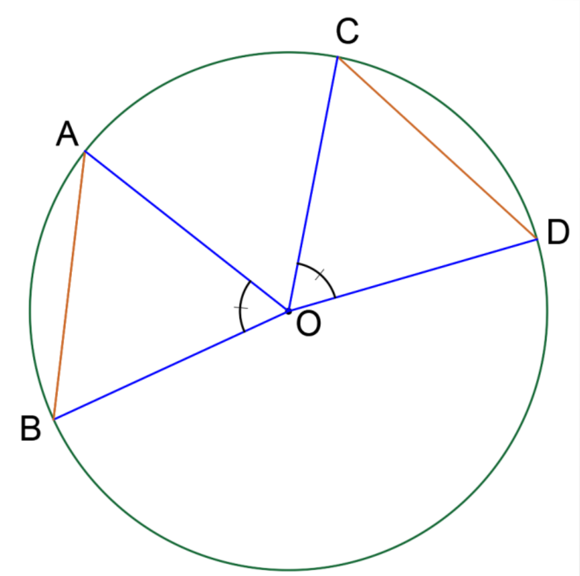
i) Angle subtended by an arc of a circle.

ii) Cyclic quadrilateral.

Angle subtended by an arc of a circle.

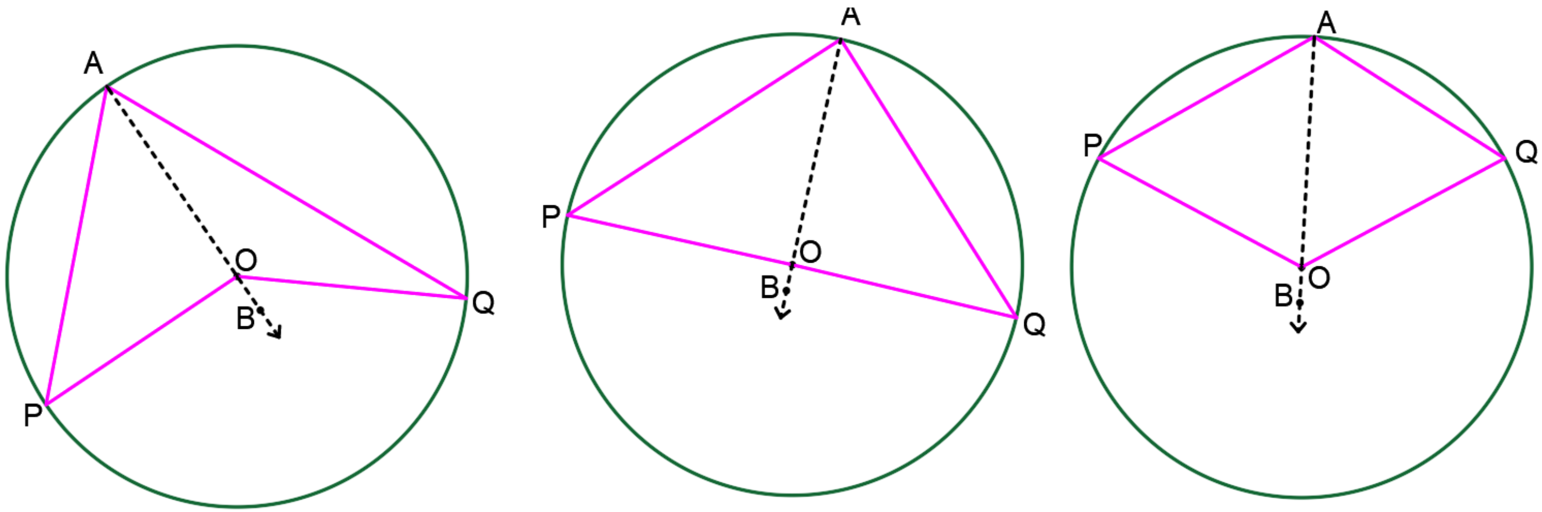


If two chords of a circle are equal, then their corresponding arcs are congruent and conversely, if two arcs are congruent, then their corresponding chords are equal.



Congruent arcs (or equal arcs)of a circle subtend equal angles at the centre.

Theorem 10.8: *The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle*.



Given: An arc PQ of a circle subtending angles POQ at the centre O and PAQ at a point A on the remaining part of the circle.

Consider the three different cases

i) arc PQ is a minor arc

ii) arc PQ is a semicircle

iii) arc PQ is a major arc.

To prove:- $\angle POQ = 2\angle PAQ$.

Construction:- Join AO and extend it to a point B.

Proof:- In all the cases

OA = OP (radii)

$\angle OAP = \angle OPA$ (angles opposite to equal sides)

$\angle POB = \angle OAP + \angle OPA$ (exterior angle of a triangle is equal to the sum of its interior opposite angles.)

$\angle POB = 2\angle OAP$. --------------- (1)

similarly

$\angle QOB = 2\angle OAQ$ ------------(2)

Adding (1) and (2) we get

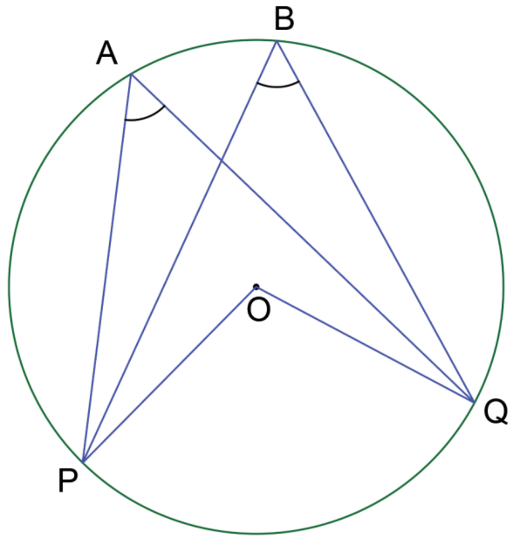
$\angle POB + \angle QOB = 2\angle OAP + 2\angle OAQ$

$\underline{\angle POQ = 2\angle PAQ}$

When PQ is a major arc

reflex angle $POQ = 2\angle PAQ$

Theorem 10.9: *Angles in the same segment of a circle are equal.*



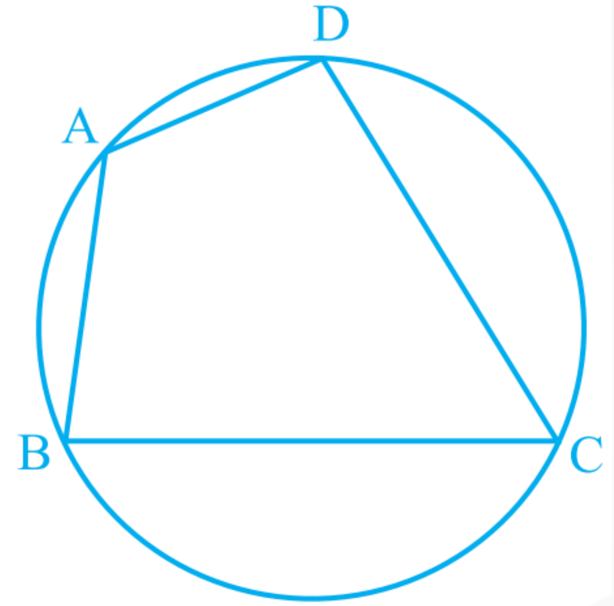
<u>∠PAQ = ∠PBQ</u>

Angle in a semicircle is a right angle.

Theorem 10.10: If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment , the four points lie on a circle.

CYCLIC QUADRILATERALS

A quadrilateral is called cyclic, if all the four vertices of it lie on a circle.

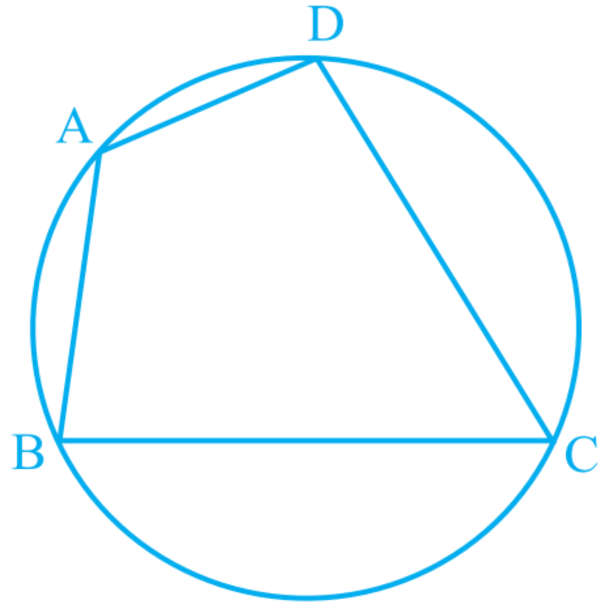


Theorem 10.11 The sum of either pairs of opposite angles of a cyclic quadrilateral is 180°.

If ABCD is a cyclic quadrilateral, then

$\angle A + \angle C = \angle B + \angle D = 180°$.

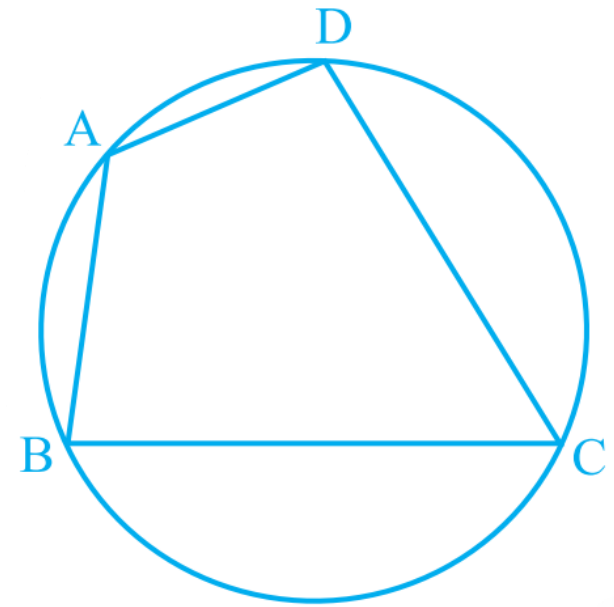
Theorem 10.12: If the sum of a pair of opposite angles of a quadrilateral is 180°, the quadrilateral is cyclic.



If in quadrilateral ABCD, if $\angle A + \angle C = 180°$ or $\angle B + \angle D = 180°$, then ABCD is a cyclic quadrilateral.

Activity.

Draw several cyclic quadrilaterals of different sides and name each of these as ABCD. Measure the angles and write your observations in the following table. What do you infer from the table?



S. No. of Quadrilateral	∠A	∠B	∠C	∠D	∠A+∠C	∠B+∠D
1.						
2.						
3.						
4.						
5.						
6.						

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